Approximate Shortest Distance Computing: A Query-Dependent Local Landmark Scheme

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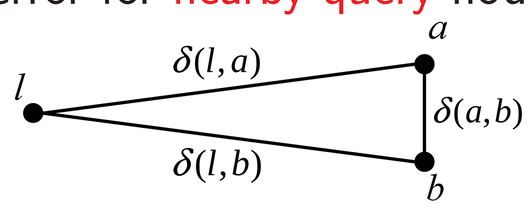
Query-Independent Global Landmark Embedding

- Given graph $G(V, E, W)$, query $q = (a, b)$, landmark set $S = \{l_1, \ldots, l_k\} \subseteq V$,

$$\widetilde{\delta}(a, b) = \min_{l_i \in S}\{\delta(l_i, a) + \delta(l_i, b)\}.$$

- Draws large error for nearby query nodes:

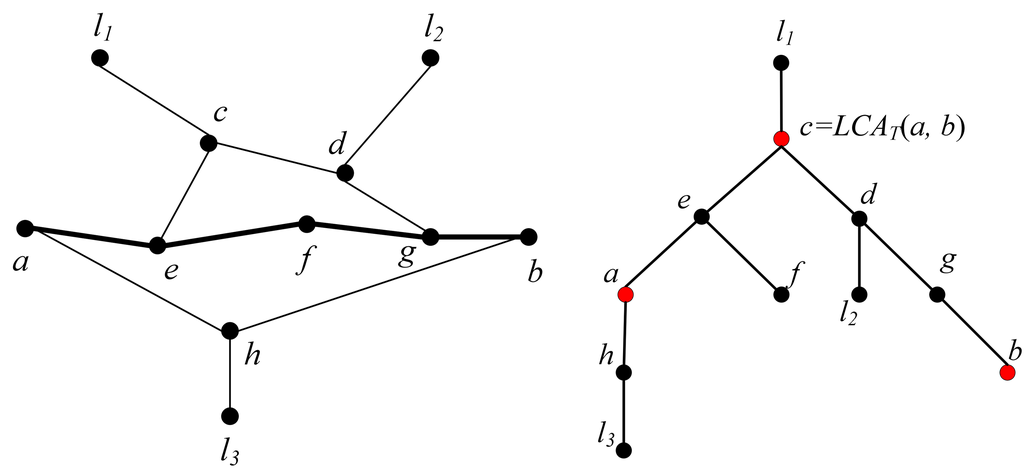


$$\widetilde{\delta}(a,b) = \delta(l,a) + \delta(l,b) >> \delta(a,b)$$

SPT Based Local Landmark Function

- T_l: the shortest path tree of l.
- $LCA_{T_l}(a, b)$: least common ancestor of a and b,

$$L_{ab}(S) = \arg \min_{r \in \{LCA_{T_l}(a,b) | l \in S\}} \{\delta(r, a) + \delta(r, b)\}.$$



$$\begin{aligned}\widetilde{\delta}^L(a, b) &= \delta(c, a) + \delta(c, b) \\ &= \delta(l_1, a) + \delta(l_1, b) - 2\delta(l_1, c)\end{aligned}$$

Accuracy and Complexity

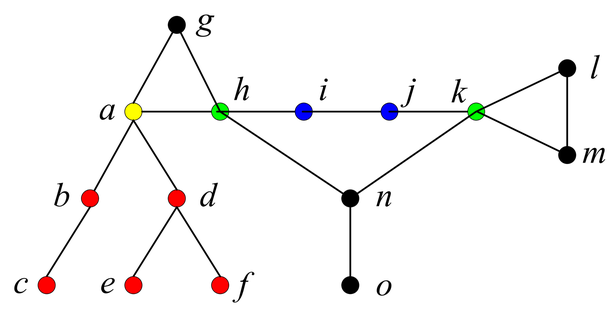
- Accuracy: $\forall a, b \in V$,

$$\delta(a, b) \leq \widetilde{\delta}^L(a, b) \leq \widetilde{\delta}(a, b).$$

- Complexity:
  - LCA Query Time: $O(1)$;
  - Online Query Time: $O(|S|)$;
  - Offline Embedding Space: $O(|S||V|)$;
  - Offline Embedding Time$O(|S||V| \log(|V|))$;
  - SAME complexities as global landmark embedding.

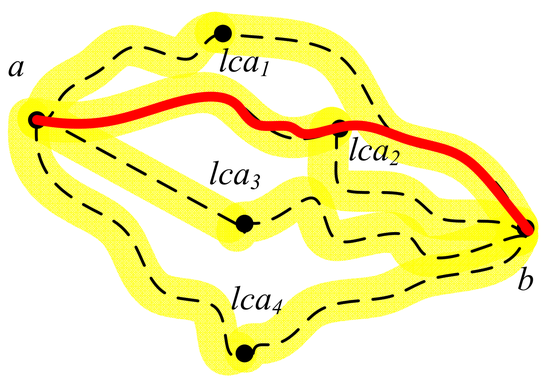
Lossless Graph Compression: Index Size Reduction

- Tree node
  - Map to the entry node.
  - Remove the tree node.
- Chain node
  - Map to two end nodes.
  - Remove the chain node.



Local Search: Accuracy Improvment

- Connect two query nodes to local landmarks through the shortest paths.
- Expand each node to include its c-hop neighbors.
- The expanded nodes may form shortcuts which provide tighter distance estimation.



Experimental Results

	SlashDot	Google	Youtube	Flickr	NYRN	USARN
	Average Relative Error					
GLS	0.6309	0.5072	0.6346	0.5131	0.1825	0.1121
LLS	0.1423	0.0321	0.0637	0.0814	0.0246	0.0786
LS	0.0000	0.0046	0.0009	0.0001	0.0071	0.0090
	Query Time(ms)					
GLS	0.002	0.005	0.008	0.009	0.006	0.020
LLS	0.006	0.021	0.015	0.014	0.036	0.067
LS	0.158	2.729	2.818	4.735	0.681	58.289
	Index Size(MB)					
GLS	6.2	57.9	90.7	124.7	21.2	1915.8
LLS	10.4	122.7	103.2	156.1	85.3	4424.6
LS	16.4	159.7	135.9	303.9	89.6	4623.6

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